Dynamic Programming

1 Problems

Problem 1: Shortest Path Counting

Find the number of the shortest paths from intersection A to intersection B in a city with perfectly horizontal streets and vertical avenues shown in this map.

need figure here

Problem 2: Blocked Paths

Find the number of different shortest paths from point A to point B in a city with perfectly horizontal streets and vertical avenues as shown in the map below. No path can cross the fenced off area shown in grey in the figure.

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Problem 3: Maximum Sum Descent

Some positive integers are arranged in a triangle like the one shown in below. Design an algorithm (more efficient than an exhaustive search, of course) to find the largest sum in a descent from its apex to the base through a sequence of adjacent numbers, one number per each level.

			②			
		5		④		
	3		4		⑦	
1		6		⑨		6

Table 1: Triangle of numbers with the maximum-sum path shown by the circles

Problem 4: Picking Up Coins

Some coins are spread in the cells of an n x m board, one coin per cell. A robot, located in the upper left cell of the board, needs to collect as many of the coins as possible and bring them to the bottom right cell. On each step, the robot can move either one cell to the right or one cell down from its current location. When the robot visits a cell with a coin, it picks up that coin. Devise an algorithm to find the maximum number of coins the robot can collect and a path it needs to follow to do this.

Problem 5: Palindrome Counting

In how many different ways can the palindrome
WAS IT A CAT I SAW
be read in the diamond-shaped arrangement shown below? You may start at any W and go in any direction on each stepup, down, left, or rightthrough adjacent letters. The same letter can be used more than once in the same sequence.

						W						
					W	A	W					
				W	A	S	A	W				
			W	A	S	I	S	A	W			
		W	A	S	I	T	I	S	A	W		
	W	A	S	I	T	A	T	I	S	A	W	
W	A	S	I	T	A	C	A	T	I	S	A	W
	W	A	S	I	T	A	T	I	S	A	W	
		W	A	S	I	T	I	S	A	W		
			W	A	S	I	S	A	W			
				W	A	S	A	W				
					W	A	W					
						W						

Table 2: Letter Arrangement

Problem 6: Pile Splitting

(a) Given n counters in a pile, split the counters into two smaller piles and compute the product of the numbers of the counters in the two piles obtained. Continue to split each pile into two smaller piles and to compute the products until there are n piles of size one. Once there are n piles, sum all the products computed. How should one split the piles to maximize the sum of the products? What is this maximal sum equal to?

(b) How does the solution to the puzzle change if we are to compute the sum of the numbers of the counters in the two piles obtained after every split and have a goal of maximizing the total of such sums?

2 Challenge Problems