Iterative Improvement

1 Problems

Problem 1: Lemonade Stand Placement

Five friends – Alex, Brenda, Cathy, Dan, and Earl – want to set up a lemonade stand. They live at the locations denoted by letters A, B, C, D, and E on the map below. At which street intersection should they place their stand to minimize the distance to their homes? Assume that they measure the distance by the total number of blocks – horizontally and vertically – from their homes to the stand.

need figure here

Problem 2: Positive Changes

Given an m x n table of real numbers, is there an algorithm to make all the row sums and column sums nonnegative by changing the signs of all the numbers in any row or column as the only operation allowed for the algorithm?

Problem 3: Heads Up

There are n coins in a line, heads and tails in random order. On each move, one can turn over any number of coins laying in succession. Design an algorithm to turn all the coins heads up in the minimum number of moves. How many moves are required in the worst case?

Problem 4: Parliament Pacification

In a parliament, each member has at most three enemies. (We assume that enmity is always mutual.) True or false: one can always divide the parliament into two chambers in such a way that no parliamentarian has more than one enemy in his or her chamber?

Problem 5: King Arthur's Round Table

King Arthur wants to seat $n > 2$ knights around his Round Table so that none of the knights is seated next to his enemy. Show how this can be done if the number of friends for each knight is not smaller than $n/2$. You may assume that friendship and enmity are always mutual.

Problem 6: Hitting a Moving Target

A computer game has a shooter and a moving target. The shooter can hit any of $n > 1$ hiding spots located along a straight line in which the target can hide. The shooter can never see the target; all he knows is that the target moves to an adjacent hiding spot between every two consecutive shots. Design an algorithm that guarantees hitting the target or prove that no such algorithm exists.

2 Challenge Problems

Problem 7: Candy Sharing

In a kindergarten, there are n children sitting in a circle facing their teacher in the center. Each child initially has an even number of candy pieces. When the teacher blows a whistle, each child simultaneously gives half of his or her candy pieces to the neighbor on the left. Any child who ends up with an odd number of pieces is given another piece by the teacher. Then the teacher blows her whistle again, unless all the children have the same number of candies, in which case the game stops. Can this game go on forever or will it eventually stop to let the children go on with their lives?