Representation Change

1 Topics

- State Machines
- Graphs
- Change of Base / Modular Arithmetic
- Basic Combinatorics?

2 Problems

Problem 1: Missionaries and Cannibals

Three missionaries and three cannibals must cross a river. Their boat can only hold two people, and it cannot cross the river by itself with no people on board. Each missionary and each cannibal can row the boat. If present, missionaries cannot be outnumbered by cannibals. How can all six get across the river with the fewest crossings?

Problem 2: Penny Distribution Machine

A "machine" consists of a row of boxes. To start, one places n pennies in the leftmost box. The machine

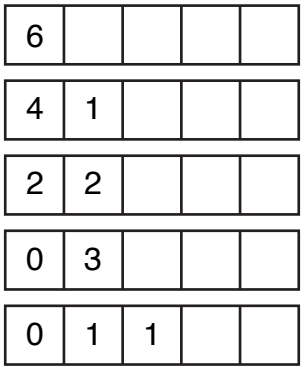

Figure 1: Example of penny distribution.

then redistributes the pennies as follows. On each iteration, it replaces a pair of pennies in one box with a single penny in the next box to the right. The iterations stop when there is no box with more than one coin. For example, Figure 1 shows the work of the machine in distributing six pennies by always selecting a pair of pennies in the leftmost box with at least two coins.

(a) Does the final distribution of pennies depend on the order in which the machine processes the coin pairs?

(b) What is the minimum number of boxes needed to distribute n pennies?

(c) How many iterations does the machine make before stopping?

3 Challenge Problems

Problem 3: Four Alternating Knights

There are four knights on a 3×3 chessboard: the two white knights are at the two bottom corners, and the

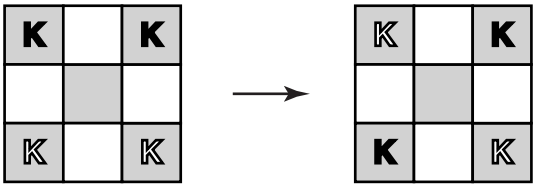

Figure 2: Desired Goal of the Four Alternating Knights puzzle.

two black knights are at the two upper corners of the board. Find the shortest sequence of moves to achieve the position shown on the right of Figure 2 or prove that no such sequence exists. Of course, no two knights can ever occupy the same square of the board.

Problem 4: Different Pairings

A kindergarten teacher has to arrange $2n$ children in n pairs for daily walks. Design an algorithm for this task so that for $2n - 1$ days no pair would be the same.

Problem 5: Turning on a Light Bulb

A light bulb is connected to n switches in such a way that it lights up only when all the switches are closed. Each switch is controlled by a push button; pressing the button toggles the switch, but there is no way to know the state of the switch. Design an algorithm to turn on the light bulb with the minimum number of button pushes needed in the worst case.

Problem 6: Poisoned Wine

An evil king is informed that exactly one of his 1000 wine barrels has been poisoned. The poison is so potent that a miniscule amount of it, no matter how diluted, kills a person on the 30th day after consumption. The king is prepared to sacrifice 10 of his slaves to determine the poisoned barrel.

(a) Can this be done before a feast scheduled in 5 weeks?

(b) Can the king achieve his goal with just 8 slaves?